Equations (3.13) and (3.14) make it possible to write an equation for the jump in F_j^i in the form

$$[F_j^i] = h^i \rho F_j^k n_k,\ h^i = -\frac{1}{\rho G}[v^i],$$

after which we shall represent the remaining equations (3.12) in the form

$$\rho G\left\{[\varepsilon] - \frac{1}{2}\left(\sigma_i^{k+} + \sigma_i^{k-}\right) h^i n_k\right\} + [q^k]\, n_k = 0, \tag{3.15}$$
$$[\sigma^{ik}]n_k - (\rho G)^2 h^i = 0,\ \rho G[W_{ij}] = 0,\ \rho G[\chi] = 0.$$

On a contact discontinuity (G = 0) $[v^i] = [\sigma^{ik}]n_k = [q^k]n_k = 0$, while the quantities $[W_{ij}]$, $[\chi]$, $[\varepsilon]$ and $[F_j^i]$ are arbitrary. In the case of a shock wave ($G \neq 0$), it follows from (3.15) that the symmetrical part W_{ij} of the plastic gradient and the strengthening parameter χ are continuous, while the remaining quantities are discontinuous.

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ALLOWANCE FOR DIFFERENCES IN STRAIN RESISTANCE IN THE CREEP OF ISOTROPIC AND ANISOTROPIC MATERIALS

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UDC 539.3

The behavior of many light alloys and also polymer, composite, and other materials under creep conditions is characterized by differences in strain resistance. This property usually manifests itself in conventional tensile, compressive, and torsional tests.

The classic creep theory for isotropic media, based on the Mises number, does not account for differences in strain resistance. It does not distinguish between tensile and compressive strain resistance characteristics and admits the possibility of analytical description of shearing strain on the basis of the characteristics determined in tensile tests in spite of the fact that it differs fundamentally from linear strain. Equal tensile, compressive, or shearing strength characteristics are ascribed to materials whose creep is satisfactorily described within the framework of the above model. In the opposite case, differences in resistance to these two or three types of strain are contemplated. Generally, the tensile, compressive, and shearing strain resistance characteristics should obviously be considered as three mutually independent characteristics of materials.

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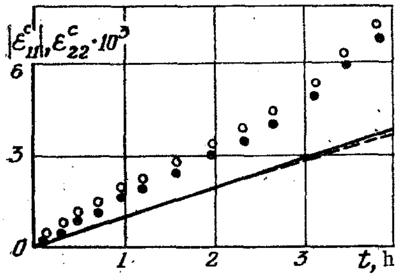

Fig. 1.

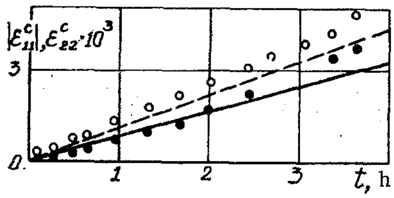

Fig. 2.

At present, there are several approaches used in describing the creep of isotropic media with varying strain resistance. Some limited surveys of these approaches are found in [1, 2].

Most of the proposed theories deal with the behavior of materials characterized only by different tensile and compressive characteristics. The differences in strain resistance are taken into account in various ways: by using the signs and magnitudes of the principal stresses, the first invariant of the stress tensor, or the third invariant of the stress deviator. Comparison between the data predicted by these theories and the experimental results for complex stressed states indicates that they are in satisfactory agreement while it does not favor any particular theory [1]. In this case, the use of simpler physical equations is probably justified.

The creep of incompressible media characterized by varying resistance with respect to all three types of strain was considered in [2, 3]. This approach is based on the concept of a potential which depends on the quadratic and cubic invariants of the stress deviator. The associated flow law is used in this case. Subsequently, this approach has been extended [4] also to orthotropic media, where the principal directions of anisotropy coincide with the principal axes of the stress tensor.

The type of relationship between strain and stress is determined by the properties of the material. The physical state of an anisotropic medium can be described by a number of tensors of different ranks. The choice of the anisotropy tensors is fairly arbitrary and is determined by the feasibility of processing the experimental results. It is very convenient to use invariants comprising the stress tensor and a certain number of these anisotropy tensors. The quadratic invariant is insufficient for materials with varying strain resistance, and odd invariants must be used. It will be shown below that, in order to describe the differences in strain resistance, at least in the case of the two-dimensional stressed state, the first and the second invariants are sufficient, and there is no need for introducing the third invariant.

Assume that σ and σ_0^2 are respectively the linear and the quadratic simultaneous invariants of the stress tensor σ_{ij} and the anisotropy tensors b_{ij} and a_{ijkl}:

$$\sigma = b_{ij}\sigma_{ij}, \quad \sigma_0^2 = a_{ijkl}\sigma_{ij}\sigma_{kl}.$$

In creep theory, one usually proceeds from the assumption that there exists a potential whose equation of surface is assumed to be

$$f = \sigma_e^2 - \varphi^2(\dot{\varepsilon}_0^c) = 0. \tag{1}$$

Here σ_e is the equivalent stress ($\sigma_e \geqslant 0$), and $\dot{\varepsilon}_0^c$ is the equivalent rate of creep strain, which, multiplied by σ_e, yields the specific energy dissipation rate:

$$W = \sigma_{ij}\dot{\varepsilon}_{ij}^c, \text{ i.e.,}$$
$$\dot{\varepsilon}_0^c\sigma_e = W; \tag{2}$$

where $\dot{\varepsilon}_{ij}^c$ are the components of the creep rate tensor. The different ways of defining the potential correspond to different concepts of the equivalent stress.

For classic anisotropic media, the equivalent stress σ_e is usually identified with σ_0 [5], i.e., it is assumed that

$$\sigma_e = \sigma_0. \tag{3}$$

For materials with varying resistance to strain, we write

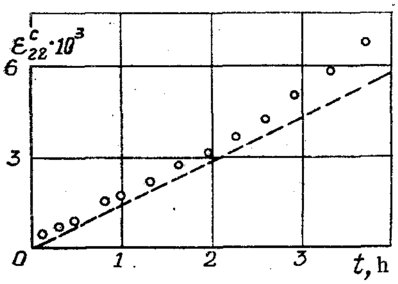

Fig. 3.

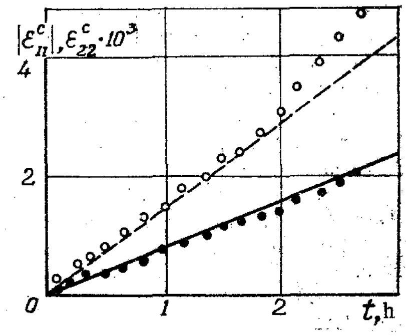

Fig. 4.

$$\sigma_e = \sigma_0 + \sigma. \quad (4)$$

Relationship (4) comprises expression (3) as a particular case.

The components of the creep rate tensor $\dot{\varepsilon}_{ij}^c$ are determined by the associated flow law [6] $\dot{\varepsilon}_{ij}^c = \lambda \partial f/\partial \sigma_{ij}$, where λ is a certain scalar factor. Then, by taking into account the relationships

$$\partial\sigma_0/\partial\sigma_{ij} = a_{ijkl}\sigma_{kl}/\sigma_0,\ \partial\sigma/\partial\sigma_{ij} = b_{ij},\ \partial f/\partial\sigma_0 = 2\sigma_e,\ \partial f/\partial\sigma = 2\sigma_e$$

we obtain the physical equations

$$\dot{\varepsilon}_{ij}^c = 2\lambda\sigma_e (a_{ijkl}\sigma_{kl}/\sigma_0 + b_{ij}).$$

By using these relationships and forming the contraction $\sigma_{ij}\dot{\varepsilon}_{ij}^c$, we obtain

$$2\lambda\sigma_e^2 = W. \quad (5)$$

Comparing Eqs. (2) and (5), we find $2\lambda\varepsilon_e = \dot{\varepsilon}_0^c$. The type of the function $\dot{\varepsilon}_0^c$ is determined from (1). Actually, $\sigma_e = h(\dot{\varepsilon}_0^c)$. There evidently also exists the inverse relationship $\dot{\varepsilon}_0^c = v(\sigma_e)$, which determines the creep curves for a uniaxial stressed state.

We note that

$$v(0) = 0. \quad (6)$$

The function $v(\sigma_e)$ can also be used in one of the following forms: the power relationship $v(\sigma_e) = \sigma_e^n$, the hyperbolic sine law $v(\sigma_e) = \mathrm{sh}(\sigma_e/A)$, and the exponential relationship $v(\sigma_e) = \exp(\sigma_e/A)$. Although condition (6) is not satisfied in the latter relationship, this presentation is often used because of its simplicity. Thus, the physical equations in the theory of creep of anisotropic media with varying resistance to strain have the following form:

$$\dot{\varepsilon}_{ij}^c = v(\sigma_e)(a_{ijkl}\sigma_{kl}/\sigma_0 + b_{ij}).$$

The material constants b_{ij} and a_{ijkl} in these relationships form tensors, which, with changes in the coordinate system, are transformed by means of the appropriate equations of tensor algebra. In view of the symmetry of the stress tensor σ_{ij}, it can be considered without loss of generality that the tensors b_{ij} and a_{ijkl} satisfy the symmetry conditions

$$b_{ij} = b_{ji},\ a_{ijkl} = a_{jikl} = a_{ijlk} = a_{klij},$$

which reduce the number of the different constants b_{ij} to six and the number of the constants a_{ijkl} to 21.

For materials which are not less symmetric than orthotropic materials, the physical relationships are written in the following form in a coordinate system whose axes coincide with the principal directions of anisotropy:

$$\dot{\varepsilon}_{11}^c = v(\sigma_e)\left(\frac{a_{1111}\sigma_{11} + a_{1122}\sigma_{22} + a_{1133}\sigma_{33}}{\sigma_0} + b_{11}\right), \quad (7)$$

$$\dot{\varepsilon}_{12}^c = 2v(\sigma_e)\frac{a_{1212}\sigma_{12}}{\sigma_0}\ (1, 2, 3),$$

$$\sigma_0^2 = a_{1111}\sigma_{11}^2 + 2a_{1122}\sigma_{11}\sigma_{22} + 2a_{1133}\sigma_{11}\sigma_{33} + 2a_{2233}\sigma_{22}\sigma_{33} + 4a_{1212}\sigma_{12}^2 +$$

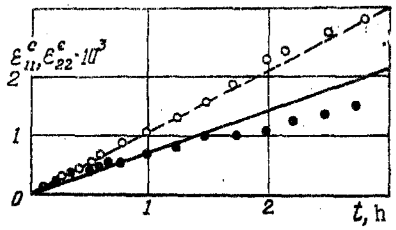

Fig. 5.

$$+4a_{1313}\sigma_{13}^2 + 4a_{2323}\sigma_{23}^2 + a_{2222}\sigma_{22}^2 + a_{3333}\sigma_{33}^2,$$
$$\sigma = b_{11}\sigma_{11} + b_{22}\sigma_{22} + b_{33}\sigma_{33}.$$

We shall demonstrate the use of Eqs. (7) for describing creep in a two-dimensional stressed state. We use the experimental data [4, 5] obtained for tubular specimens of D16T material at 250°C, which were subjected to internal pressure with simultaneous application of an axial tensile or compressive load. The relationship between the creep rate tensor and the stress tensor is in this case given by

$$\dot{\varepsilon}^c_{11} = (\sigma + \sigma_0)^n \left(\frac{a_{1111}\sigma_{11} + a_{1122}\sigma_{22}}{\sigma_0} + b_{11}\right)(1, 2),$$
$$\sigma_0^2 = a_{1111}\sigma_{11}^2 + 2a_{1122}\sigma_{11}\sigma_{22} + a_{2222}\sigma_{22}^2,$$
$$\sigma = b_{11}\sigma_{11} + b_{22}\sigma_{22}.$$

The creep curves for the uniaxial stressed state [5] are described by the expressions

$$\dot{\varepsilon}^c_{11} = A_1^k |\sigma_{11}|^{N-1}\operatorname{sign}(\sigma_{11}),\ \dot{\varepsilon}^c_{22} = -\nu_2^k \dot{\varepsilon}^c_{11}(1, 2),\quad k = 1, 2.$$

The value k = 1 corresponds to extension, while k = 2 corresponds to compression. The following material constants are known [5]: N = 7.5, $A_1^1 = 4.32\cdot 10^{-11}$, $A_1^2 = 2.24\cdot 10^{-11}$, and $A_2^1 = 9.40\cdot 10^{-11}$ $(\mathrm{mm}^2/\mathrm{kg})^{N-1}\mathrm{h}^{-1}$. Using the experimental data on tubular specimens tested under axial compression and on specimens subjected to internal pressure only given in the above paper, we determine the coefficients $\nu_2^2 = 0.53$ and $\nu_1^1 = 0.33$. These experimental constants are sufficient for determining the anisotropy parameters in the physical relationships under consideration: n = N − 1 = 6.5,

$$a_{1111} = \left[(A_1^1)^{1/(n+1)} + (A_1^2)^{1/(n+1)}\right]^2/4 = 1.58\cdot 10^{-3}\ (\mathrm{kg/mm^2})^{-2n/(n+1)}\mathrm{h}^{-2/(n+1)},$$
$$b_{11} = \left[(A_1^1)^{1/(n+1)} - (A_1^2)^{1/(n+1)}\right]/2 = 1.74\cdot 10^{-3}\ (\mathrm{kg/mm^2})^{-n/(n+1)}\mathrm{h}^{-1/(n+1)},$$
$$a_{2222} = \left[(A_1^2)^{1/(n+1)}\nu_2^2 - (A_2^1)^{1/(n+1)}\right]^2 a_{1111}/\left[b_{11} + (A_2^1)^{1/(n+1)}\nu_1^1 - \sqrt{a_{1111}}\right]^2 =$$
$$= 2{,}03\cdot 10^{-3},\quad b_{22} = (A_2^1)^{1/(n+1)} - \sqrt{a_{2222}} = 9{,}57\cdot 10^{-4},\quad a_{1122} = -\sqrt{a_{2222}}\left[b_{11} + (A_2^1)^{1/(n+1)}\nu_1^1\right] = -7{,}63\cdot 10^{-4}.$$

Consider five loading programs: 1) $\sigma_{11} = -10.98$, $\sigma_{22} = 5.49$; 2) $\sigma_{11} = -8.06$, $\sigma_{22} = 8.06$; 3) $\sigma_{11} = 7$, $\sigma_{22} = 14$; 4) $\sigma_{11} = -3.76$, $\sigma_{22} = 11.28$; 5) $\sigma_{11} = 12.4$ kg/mm², $\sigma_{22} = 12.4$ kg/mm².

Figures 1-5 show for each case respectively the variation of the absolute values of the axial ε^c_{11} (solid curve) and tangential ε^c_{22} (dashed curve) strains with the time t. The experimental data are indicated by solid (axial strain) and open (tangential strain) circles. Thus, the agreement between the theoretical and the experimental results is entirely satisfactory.

TABLE 1

No.	σ_{11}, kg/mm²	σ_{12}, kg/mm²	$\dot{\varepsilon}^c\cdot 10^4$, h^{-1}	$\dot{\gamma}^c\cdot 10^4$, h^{-1}	$\dot{\varepsilon}^c_{11}\cdot 10^4$, h^{-1}	$2\dot{\varepsilon}^c_{12}\cdot 10^4$, h^{-1}
1	12,23	7,06	0,36	0,7	0,37	0,73
2	−13,08	7,55	−0,32	0,6	−0,37	0,82
3	15,24	5,08	0,49	0,54	0,46	0,53
4	−17,56	4,2	−0,5	0,39	−0,45	0,40
5	6,62	9,23	0,19	0,9	0,24	1,09
6	−6,87	9,58	−0,14	0,93	−0,21	1,18

By using the equalities

$$b_{11} = b_{22} = b_{33},\ a_{1111} = a_{2222} = a_{3333},\ a_{1212} = a_{1313} = a_{2323},$$
$$a_{1122} = a_{1133} = a_{2233},\ a_{1111} = 2a_{1212} + a_{1122},$$

in relationships (7), we arrive at the physical relationships of creep theory for isotropic media with unequal tensile, compressive, and shearing strain characteristics within the framework of the above approach. If a material has different resistance characteristics with respect to tensile and compressive strain only, one ordinarily uses the condition $4\,a_{1212} = 3\,a_{1111}$ as well. In this case, σ_0 coincides with the stress intensity with an accuracy to a constant.

It should be noted that the proposed physical equations describe a number of effects which are known to hold for isotropic media on the basis of experiments: dependence of the creep process on hydrostatic pressure [7, 8], departure from similarity of the creep rate and stress deviators [7-9], compressibility [7, 8], existence of axial creep under pure shearing conditions [2, 9], dilation [8], etc.

We shall demonstrate the use of these physical relationships for describing creep in a complex stressed state in isotropic materials with mutually independent tensile, compressive, and torsional characteristics. We shall consider the experimental data obtained for tubular specimens under the simultaneous action of a tensile or compressive force and a torque [2, 3]. The material is AK4-1T alloy, tested at 200°C. In this case, the assumption concerning the interdependence of the material's modes of behavior under tensile, compressive, and torsional loads and the use of the equations of classic creep theory produce results which differ from experimental data by as much as 150% [3].

The proposed physical relationships are written in the following form:

$$\dot{\varepsilon}^c_{11} = v(\sigma_e)\,(a_{1111}\sigma_{11}/\sigma_0 + b_{11}),$$
$$\dot{\varepsilon}^c_{12} = 2v(\sigma_e)\,a_{1212}\sigma_{12}/\sigma_0,$$
$$\sigma_0^2 = a_{1111}\sigma_{11}^2 + 4a_{1212}\sigma_{12}^2,\quad \sigma = b_{11}\sigma_{11}.$$

In processing the creep curves given in [2], it was assumed that $v(\sigma_e) = (\sigma + \sigma_0)^n$, and it was found that $n = 8$, $a_{1111} = 6.15\cdot10^{-4}$, $a_{1212} = 5.57\cdot10^{-4}$ $(\text{kg/mm}^2)^{-2n/(n+1)}h^{-2/(n+1)}$, and $b_{11} = 9.54\cdot10^{-4}$ $(\text{kg/mm}^2)^{-n/(n+1)}h^{-1(n+1)}$. Table 1 provides the experimental $\dot{\varepsilon}^c$ and $\dot{\gamma}^c$ and the theoretical $\dot{\varepsilon}^c_{11}$ and $2\dot{\varepsilon}^c_{12}$ values of the strain rates for different combinations of tensile (compressive) and torsional loads. The experimental data were borrowed from [3]. We note the satisfactory agreement between the theoretical and experimental values of the strain rate. Similar results can also be obtained by using the more complex relationships involving the third invariant of the stress deviator [2, 3].

Thus, the proposed physical relationships, presented in the fairly simple and convenient tensor invariant form, can be used for analyzing the creep of various structural elements made of anisotropic materials characterized by differences in strain resistance.

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